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1.

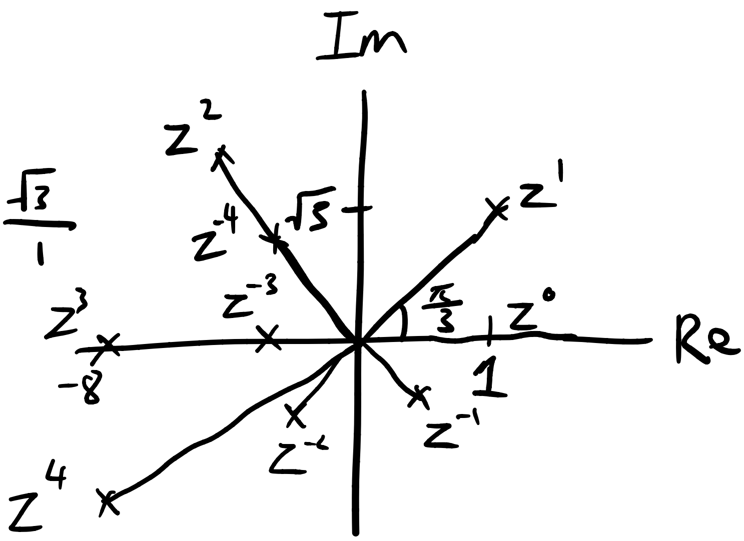
$z = 1+\sqrt{3}i$

$r = \sqrt{1^2+\sqrt{3}^2}$

$= 2$

$\theta = \tan^{-1}\frac{\sqrt{3}}{1}$

$= \frac{\pi}{3}$



$z = 2e^{\frac{\pi}{3}i}$

$z^2 = 1+2\sqrt{3}i-3$

$= -2+2\sqrt{3}i = 4e^{\frac{2\pi}{3}i}$

$z^3 = (-2+2\sqrt{3}i)(1+\sqrt{3}i)$

$= -2+2\sqrt{3}i-2\sqrt{3}i+6i^2$

$= -8 \qquad = 8e^{i\pi}$

$z^4 = -8-8\sqrt{3}i \quad = 16e^{i\frac{4\pi}{3}}$

$z^{-1} = \frac{1}{2}e^{-\frac{\pi}{3}i}$

$z^{-2} = \frac{1}{4}e^{-\frac{2\pi}{3}i}$

$z^{-3} = \frac{1}{8}e^{-\pi i}$

$z^{-4} = \frac{1}{16}e^{-\frac{4\pi}{3}i}$

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2. $e^{a+bi} = 1+\sqrt{3}i$ $\qquad z = 2e^{\frac{\pi}{3}i}$

$\Rightarrow z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$$e^{a+bi} = e^a \cdot e^{bi}$$
$$= e^a(\cos b + i\sin b)$$
$$= e^a\cos b + ie^a\sin b$$

$\Rightarrow e^a = 2 \qquad b = \frac{\pi}{3}$

$a = \ln 2$

$\therefore a+bi = \ln 2 + \frac{\pi}{3}i$

$e^{n(a+bi)}$

$n=1, \quad e^{\ln 2 + \frac{\pi}{3}i}$

$n=2, \quad e^{2\ln 2 + \frac{2\pi}{3}i}$

$n=3, \quad e^{3\ln 2 + \pi i}$

$n=4, \quad e^{4\ln 2 + \frac{4\pi}{3}i}$

$n=0, \quad 1$

$n=-1, \quad e^{-\ln 2 - \frac{\pi}{3}i}$

$n=-2, \quad e^{-2\ln 2 - \frac{2\pi}{3}i}$

$n=-3, \quad e^{-3\ln 2 - \pi i}$

$n=-4, \quad e^{-4\ln 2 - \frac{4\pi}{3}i}$